

## HEAD-TAIL INSTABILITY AT TEVATRON

P. M. Ivanov, J. Annala, A. Burov, V. A. Lebedev, E. Lorman, V. Ranjbar, V. Scarpine, V. Shiltsev  
FNAL, Batavia, IL 60510, USA

### Abstract

Tevatron performance suffers from a coherent transverse instability. Experimental studies and theoretical examination allow identifying the instability as a weak head-tail, driven by the short-range wake fields in presence of the space charge. Growth rates and coherent tune shifts are measured at injection of single high-intensity proton bunches using a fast strip-line pickup. Landau damping through the octupole-generated betatron tune spread for all of unstable head-tail modes has been demonstrated.

### INTRODUCTION

In order to prevent developing a transverse coherent instability for high intensity proton beam, the Tevatron lattice chromaticities are set as high as  $\xi_{x,y} \approx +8$  at the injection ( $E=150$  GeV) and  $\xi_{x,y} \approx +26$  at the collision energy ( $E=980$  GeV). Although this instability suppression looks successful, it results in a degradation of the machine performance due to reduction of beam lifetime at the injection energy. The width of the chromatic betatron tune spread is proportional to the chromaticities:  $\Delta\nu_{x,y} = 2 \left| \xi_{x,y} \cdot \Delta p / p \right|$ . Observation of the particle losses by monitors around the CDF-detector demonstrates the similar correlation with the absolute value of chromaticity with minimum at  $\xi_{x,y} = 0$ . In addition, the chromatic modulation effect can interfere in beam-beam interaction. All of the above-mentioned reasons stimulated us to investigate a driving mechanism for this instability, as well as to search for possible solutions for operation at zero chromaticities.

### INSTABILITY OBSERVATION

It was suggested that a major source of impedance is the Lambertson injection magnet, having high resistivity due to the laminations. The impedance is inversely proportional to the aperture and it can be estimated by integrating the resistance over the low frequency current pass through the laminas [2]:

$$Z_{\perp} \approx \frac{2Z_o}{\pi b^2} \cdot \frac{\mu}{\kappa} \cdot F \cdot \frac{L}{d} \quad (1)$$

where  $\kappa^2 = -4\pi i\sigma\mu\omega/c^2$ ,  $Z_o \approx 377\Omega$ ,  $\mu \approx 100$ ,  $F = 0.5 - 1.0$  is a geometry form-factor,  $L = 11.2$  m is a total length of the magnet, and  $d = 1$  mm is the lamination thickness.

Instability observation clearly has shown that the beam orbit shift inside the Lambertson magnet aperture leads to qualitative changes in the stability condition for coherent head-tail modes. This fact strongly points at a dominant contribution of the magnet to the impedance budget. Stability bounds in chromaticity space have been measured for the three beam orbit offsets (see Fig.1). The results are presented in Figure 2.

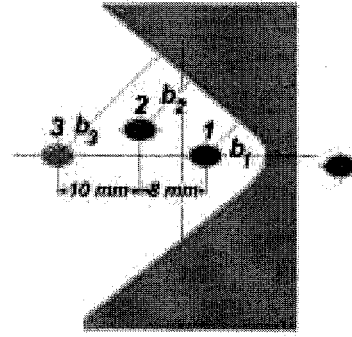


Figure.1. The Lambertson magnet impedance is estimated in accordance with Eq. (1) for three different effective distances:

1. Injection local orbit bump:  $b_1 \approx 6$  mm,  $Z_{\perp} \approx 5$  M $\Omega$ /m
2. Central regular orbit:  $b_2 \approx 9$  mm,  $Z_{\perp} \approx 1.8$  M $\Omega$ /m;
3. Local orbit bump with respect to the central orbit  $\Delta Y = -3$  mm,  $\Delta X = -10$  mm,  $b_3 \approx 18$  mm,  $Z_{\perp} \approx 0.6$  M $\Omega$ /m.

On the injection orbit, the head-tail instability is polarized in the vertical plane at positive chromaticities. Stability is limited by excitation of the quadrupole mode with longitudinal number  $l=2$  (see Fig. 3). The coherent mode with the monopole longitudinal configuration  $l=0$  limits stability in horizontal plane when  $\xi_x \approx -1$ .

Horizontal and vertical impedances of Eq. 1 are approximately equivalent, but the stability bounds for the vertical and horizontal modes are in principle different (Fig.2). A possible reason could be related to the space charge tune shifts, which are different for the two planes because of dispersion; the vertical incoherent shift is two times larger than the horizontal shift. Calculated coherent tune shifts for the first two horizontal modes are found to be comparable with the incoherent space-charge tune shift that promotes Landau damping due to a synchrotron tune spread. The vertical modes are in worse conditions because the space-charge shift is higher. At Tevatron energy  $E=150$  GeV the synchrotron tune and rms-tune spread are:  $\nu_{s0} = 1.8 \cdot 10^{-3}$ ,  $\delta\nu_s \approx 2.2 \cdot 10^{-4}$

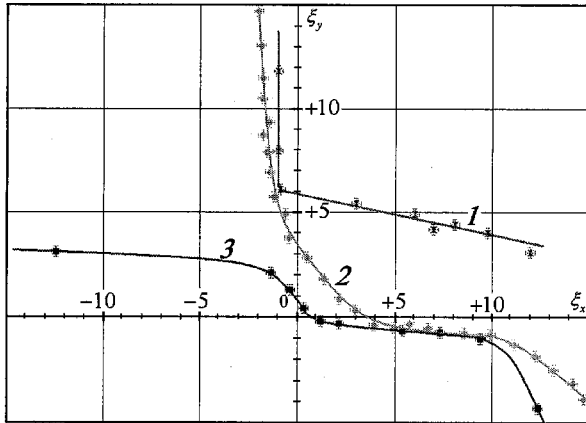


Figure 2. The stability regions for the head-tail modes in the chromaticity space. All measurements are performed with single proton bunch ( $N_{ppb} = 2.6 \cdot 10^{11}$ ). The thresholds of the excitation correspond to an increase in the coherent component of the Schottky spectrum as the chromaticities were smooth decreased.

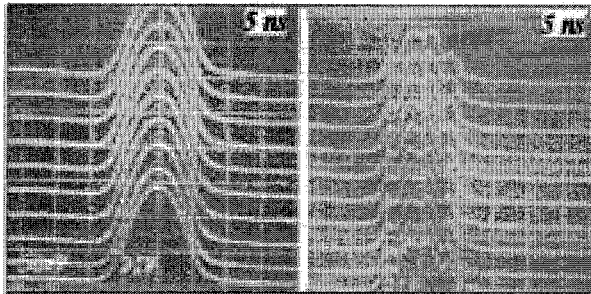


Figure 3. Longitudinal density profiles of the initial ( $N_{ppb} = 2.65 \cdot 10^{11}$ ) and remaining ( $N_{ppb} = 1.03 \cdot 10^{11}$ ) proton bunches before and after self-stabilization of the vertical instability due to the particle losses. The particles were lost in accordance with the longitudinal configuration of the coherent vertical oscillations that points qualitatively at excitation of the head-tail mode with  $l=2$ .

The Tevatron standard operational tunes  $\nu_x = 20.585$  and  $\nu_y = 20.575$  are located in vicinity of the resonance line  $\nu_x - \nu_y = 0$ . At crossing the betatron coupling resonance both the vertical and horizontal coherent modes become unstable that can be caused by a repartition of the direct space-charge tune shift between two normal betatron modes with coupling increased.

On central orbit (curve 2), a single coherent vertical mode with dipole longitudinal configuration is observed at the chromaticity threshold  $\xi_y \leq 3$ . The horizontal higher order head-tail modes are stable out of the resonance  $\nu_x - \nu_y = 0$ .

At the orbit bump  $\Delta Y = -3 \text{ mm}$  and  $\Delta X = -10 \text{ mm}$  with negative chromaticities (curve 3), the unstable mode

$l=0$  can be easily stabilized by means of octupoles. Similar stability conditions are expected after installation of planned shielding of the magnet bare lamination.

### Turn-by turn measurement technique

A fast digital oscilloscope, connected to the horizontal and vertical 1-meter long strip-line pickups, records the transverse head-tail dynamics for 2000 turns allowing clearly identify some of the coherent unstable modes. At the injection orbit bump, the coherent tune shifts and growth rates were measured with the turn-by-turn monitor system at injections of single proton bunches.

At  $N_{ppb} = 2.6 \cdot 10^{11}$  and chromaticity value of  $\xi_{x,y} \approx -2$ , the coherent mode with  $l=0$  has the following measured parameters:

$$\Delta \nu_{x,y}^{l=0} = 0.001 \pm 0.0001 \quad 1/\tau_0 = 120 \pm 5 \text{ sec}^{-1}$$

We were also able to measure chromaticity by pinging the beam and measuring the phase shift between head and tail from the turn-by turn data. The chromaticity can be extracted from this phase difference according to [4]:

$$\xi_{x,y} = -\eta \frac{\Delta \Psi_{x,y}}{\omega_0 \Delta \tau (\cos(2\pi n \nu_s) - 1)} \quad (2)$$

Here  $\Delta \Psi$  is the head-tail phase difference,  $\eta$  is the slippage factor,  $\omega_0$  is the revolution frequency, and  $\Delta \tau$  is the time length of the bunch.

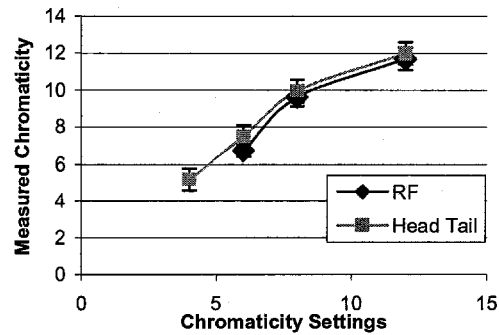


Figure 4. Comparison of the Head-Tail chromaticity measurement with the traditional RF technique.

### Computer simulation

An ensemble of macro-particles with the Gaussian longitudinal distribution has been tracked for many turns with the particle interaction with each other through the resistive wake field. An asymptotic growth rate of the particle dipole moment found in the simulations matched with the measured one leads to a certain prediction for the total impedance. At the injection bump, it comes out as  $Z_1^{\perp} \cong 4-5 \text{ M}\Omega/\text{m}$ . This impedance is calculated from the growth rate of  $120 \text{ sec}^{-1}$ . However, the actual machine impedance might be larger, since Landau damping has not been taken into account in the numerical simulations.

## SPACE CHARGE EFFECT

The space charge effect results in a non-linear incoherent detuning and plays important role in coherent head-tail dynamics. Laslett tune shifts due to electric- and magnetic-image fields are not included in our consideration because for the Tevatron performance parameters, the image terms are negligible small as compared with a contribution of the electromagnetic self-fields. For the 3D-Gaussian charge distribution the direct space charge tune shifts in maximum are given by:

$$\Delta\nu_{x,y}^{sc} = -\frac{N_{ppb} r_0 R_0}{\sqrt{2\pi} \beta^2 \gamma^3 \sigma_z} \left\langle \frac{\beta_{x,y}}{\sigma_{x,y}(\sigma_x + \sigma_y)} \right\rangle \quad (3)$$

where  $\sigma_{x,y}(s) = \sqrt{\epsilon_{x,y} \beta_{x,y}(s) + D_{x,y}^2(s) \sigma_{dp/p}^2}$  are the transverse beam sizes, the  $\langle \dots \rangle$ -operator means averaging over the machine. For  $N_{ppb} = 2.6 \cdot 10^{11}$  and  $\sigma_s = 90 \text{ cm}$ , it comes out as:

$$\Delta\nu_x^{sc} \approx -0.36 \cdot 10^{-3}, \Delta\nu_y^{sc} \approx -0.7 \cdot 10^{-3}$$

## DAMPING OF THE HEAD-TAIL MODES

At present moment, in order to work at decreased chromaticities ( $\xi_x \approx +6, \xi_y \approx +4$ ), the transverse dampers are used to prevent an excitation of the transverse instability at the multi-bunch mode of operation [6].

An universal method for damping the instability is to introduce a betatron frequency spread that is larger than the growth rates. Landau damping is effective when the following approximate condition is satisfied (Eq.4):

$$\sqrt{(\delta\nu_{x,y}^{oct})^2 + (\delta\nu_{x,y}^{sc})^2} + (l \cdot \delta\nu_s)^2 \geq |\Delta\nu_{x,y}^{sc} - \Delta\nu_{x,y}^{coh}(l)|$$

The two Tevatron regular octupole families are used to provide Landau damping for the head-tail modes:  $OZD(n=12, \beta_x > \beta_y)$ ,  $OZD(m=24, \beta_y > \beta_x)$ .

There are two sources of the octupole-driven tune spread: due to the betatron amplitudes and due to dispersion in the octupole location.

$$\delta\nu_{x,y}^\beta = \frac{1}{16\pi B\rho} \left[ J_{x,y} \sum_1^{n,m} (\bar{K}_3 \beta_{x,y}^2)_{n,m} - 2J_{y,x} \sum_1^{n,m} (\bar{K}_3 \beta_x \beta_y)_{n,m} \right] \quad (5)$$

$$\delta\nu_{x,y}^D = \frac{\sigma_{dp/p}^2}{16\pi B\rho} \sum_1^{n,m} (\bar{K}_3 \beta_{x,y} D_x^2)_{n,m} \quad (6)$$

where  $J_{x,y} = a_{x,y}^2 / \beta_{x,y}$  are single particle Courant-Snyder invariants and

$$\bar{K}_3(n,m) = \mathcal{J}_{n,m}(\text{Amps}) \cdot \int_0^{L_0} \frac{\partial^3 B_y}{\partial x^3} ds / 1 \text{ Amp} = 616 \cdot \mathcal{J}_{n,m} [T/m^2]$$

are the normalized octupole strengths with  $\mathcal{J}_n, \mathcal{J}_m$  as the OZF- and OZD-family octupole currents. On the central orbit, damping the vertical mode  $l=1$  required currents

$\mathcal{J}_{OZD} \approx 4.2 \text{ A}$  and  $\mathcal{J}_{OZF} = 0$  with the estimated tune spreads as:  $\langle \delta\nu_y^{Oct} \rangle \approx 0.28 \cdot 10^{-3}, \langle \delta\nu_x^{Oct} \rangle \approx 1 \cdot 10^{-4}$ .

At the chromaticity of  $\xi_{x,y} \approx -2$  the coherent mode  $l=0$  has been stabilized at  $\mathcal{J}_{OZD} \approx 5 \text{ A}$  and  $\mathcal{J}_{OZF} \approx 2 \text{ A}$  with  $\langle \delta\nu_y^{Oct} \rangle \approx 0.52 \cdot 10^{-3}, \langle \delta\nu_x^{Oct} \rangle \approx 0.38 \cdot 10^{-3}$ .

In both cases the widths of betatron spectra measured by Schottky monitor are in a reasonable agreement with this calculation taking into account the contributions from the synchrotron and direct space-charge tune spreads. The octupole cubic non-linearity has the positive sign that is urgent from dynamic aperture point of view since the vertical tune is slightly above the resonance  $\nu_y = 4/7$ .

Besides, it is "right" sign to minimize the octupole strengths of the OZD-family in consequence of:

$$(\Delta\nu_y^{sc} - \Delta\nu_y^{coh}(l=0)) > 0 \quad (7)$$

In horizontal plane the incoherent and coherent tune shifts are comparable but the space-charge tune spread does not promote Landau damping without octupoles.

## CONCLUSION

The observed single-bunch head-tail instability was found to be driven mainly by the resistive impedance of laminated Lambertson magnets. To reduce that, the insertion of a thin shielding liner inside the magnet is planned. It is expected to stabilize the higher order head-tail modes at positive chromaticities and significantly reduce the growth time at negative chromaticities.

Landau damping through the octupole-generated betatron tune spread for all of the unstable head-tail modes at positive and negative chromaticities has been seen. After performing some additional experimental studies, this method has planned to be involved in the routine machine operation that will result in an enhancement of the peak and integrated luminosity.

## ACKNOWLEDGMENTS

We would like to thank V. Danilov (ORNL) for useful discussions of the instability problems and Dean Still (FNAL) for helping in machine studies.

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